

I should be able to

- evaluate powers with rational exponents, simplify algebraic expressions involving exponents, and solve problems involving exponential equations graphically and using common bases;
- describe trends based on the interpretation of graphs, compare graphs using initial conditions and rates of change, and solve problems by modelling relationships graphically and algebraically;
- make connections between formulas and linear, quadratic, and exponential relations, solve problems using formulas arising from real-world applications, and describe applications of mathematical modelling in various occupations.

Vocabulary

Grade 12 College Mathematics - Unit 1

MATHEMATICAL MODELS

1

Solve for x.

a) $3x - 5 = 7 - 3x$

$$3x + 3x = 7 + 5$$
$$\frac{6x}{6} = \frac{12}{6}$$
$$x = 2$$

c) $-6x - 10 = 3x + 8$

$$-6x - 3x = 8 + 10$$
$$\frac{-9x}{-9} = \frac{18}{-9}$$
$$x = -2$$

b) $12 + 3x = x - 14$

$$\frac{2x}{2} = \frac{-26}{2}$$
$$x = -13$$

d) $-x = x + 6$

$$-x - x = 6$$
$$\frac{-2x}{-2} = \frac{6}{-2}$$
$$x = -3$$

2

Solve for x.

a) $4(x - 3) = 3x$

$$4x - 12 = 3x$$
$$4x - 3x = 12$$
$$x = 12$$

b) $12(5 - x) = 72$

$$60 - 12x = 72$$
$$-12x = 72 - 60$$
$$-12x = 12$$
$$x = -1$$

c) $-3(2x - 5) = -x +$

$$-6x + 15 = -x + 1$$
$$-6x + x = 1 - 15$$
$$-5x = -14$$
$$x = \frac{-14}{-5}$$
$$x = \frac{14}{5}$$

3

Write each as a single power. Then, evaluate

a) $3^2 \times 3^1 \times 3^5 = 3^{2+1+5}$

$$= 3^8$$

$$= 6561$$

b) $b^4 \times b^3$, when $b = 2$

$$(b^4)(b^3) = b^{4+3}$$
$$= b^7$$
$$= 2^7$$
$$= 128$$

c) $\frac{6^5}{6^3} = 6^{5-3}$

$$= 6^2$$

$$= 36$$

d) $\frac{y^7}{y^5}$ when $y = 9$

$$\frac{y^7}{y^5} = y^{7-5}$$
$$= y^2$$
$$= (9)^2$$
$$= 81$$
$$\frac{a^6 b^2 c^{-5}}{a^4 b c^2} = a^{6-4} b^{2-1} c^{-5-2}$$
$$= a^2 b^1 c^{-7}$$
$$= (2)^2 (3)^1 (-1)^{-7}$$
$$= (4)(3)(-1)$$
$$= -12$$

e) $\frac{a^6 b^2 c^{-5}}{a^4 b c^2}$ when $a = 2$, $b = 3$ and $c = -1$

4

Simplify. Express your answer using positive exponents.

a) $x^3 y^6 x^4 y^{-3} = x^3 x^4 y^6 y^{-3}$

$$= x^7 y^3$$

b) $((-t)^3)^5 = (-t)^{15}$

c) $(4x^2)^5 \times (x^4)^{-1}$

$$= (4^5 x^{10})(x^{-4})$$
$$= 1024 x^6$$

$$\frac{a^6 b^3 c^{-5}}{a^4 b c^2} = a^{6-4} b^{3-1} c^{-5-2}$$
$$= a^2 b^2 c^{-7}$$
$$= \frac{a^2 b^2}{c^7}$$

5

Simplify.

a) $(4^{1/2})(4^{1/4}) = 4^{\frac{1}{2} + \frac{1}{4}}$

$$= 4^{\frac{2}{4} + \frac{1}{4}}$$
$$= 4^{\frac{3}{4}}$$

c) $(4^{2/5})^{3/2} = (4^{\frac{2}{5} \cdot \frac{3}{2}})$

$$= 4^{\frac{6}{10}}$$
$$= 4^{\frac{3}{5}}$$

e) $2^2 \cdot 8^{\frac{-2}{3}} = 4 \cdot (3\sqrt[3]{8})^{-2}$

$$= 4 \cdot \frac{1}{(3\sqrt[3]{8})^2}$$
$$= \left(\frac{4}{1}\right) \left(\frac{1}{4}\right)$$
$$= \frac{4}{4}$$

b) $x^{3/5} \div x^{1/5} = x^{\frac{3}{5} - \frac{1}{5}}$

$$= x^{\frac{2}{5}}$$

d) $27^{\frac{-2}{3}} = (3\sqrt[3]{27})^{-2}$

$$= 3^{-2}$$
$$= \frac{1}{3^2}$$

f) $5 \cdot 25^{\frac{1}{2}} = (5)(25^{\frac{1}{2}})$

$$= 5(2\sqrt{25})$$
$$= (5)(5)$$
$$= 25$$

6

Solve for x.

a) $2^x = 4$

$$2^x = 2^2$$
$$\therefore x = 2$$

b) $2^{x+1} = 256$

$$2^{x+1} = 2^8$$
$$\therefore x+1 = 8$$
$$x = 7$$

e) $4^3 = 2^x$

$$(2^2)^3 = 2^x$$
$$2^6 = 2^x$$
$$\therefore 6 = x$$

g) $25^{2x-9} = 5^x$

$$(5^2)^{2x-9} = 5^x$$
$$5^{2(2x-9)} = 5^x$$
$$5^{4x-18} = 5^x$$
$$4x-18 = x$$
$$4x-x = 18$$
$$3x = 18$$
$$x = 6$$

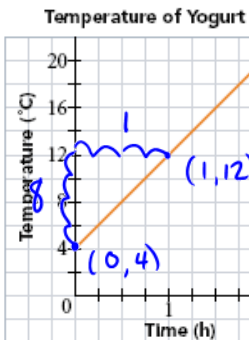
h) $2^x = 1/32$

$$2^x = 2^{-5}$$
$$\therefore x = -5$$

7

The graph shows the temperature of a container of yogurt that was left on the kitchen counter.

- a) What value is plotted on the horizontal axis? **time (s)**
- b) What value is plotted on the vertical axis? **temperature (°C)**
- c) Determine the slope of the line. Include the units in your answer.
- d) Explain the meaning of the slope.



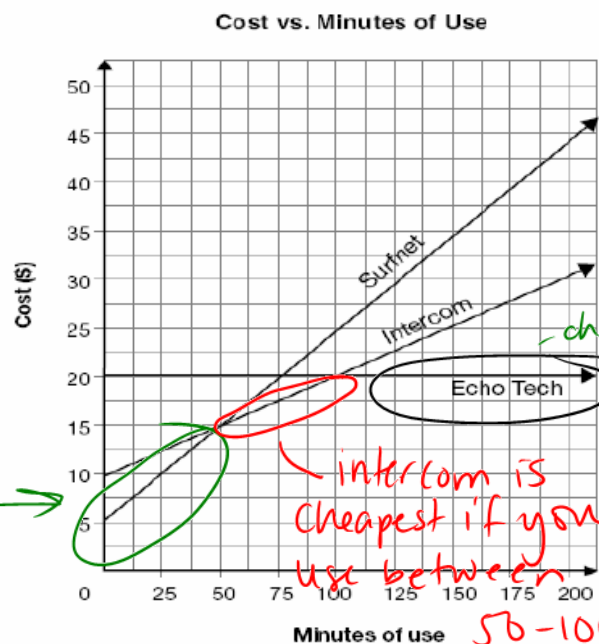
c) $\text{slope} = \frac{\text{rise}}{\text{run}}$

$$= \frac{8}{1}$$
$$= 8^\circ\text{C/h}$$

d) The slope represents the rate of change in temperature over time

8

The graph shows the relationship between total cost and minutes of use for three Internet companies. Tenisha wants to sign up with one of the companies and she wants to pay as little as possible. Her choice will depend on her minutes of use. Determine which company Tenisha should sign up with. Include details about minutes of use in your explanation.



11

Grandma's old cookie jar is a valuable antique. The graph shows how its value increases with time.

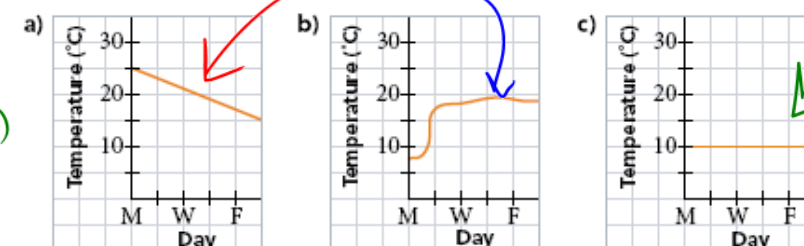
- How much is the cookie jar worth initially? *\$60*
- How much will it be worth in 10 years? *approximately \$95*
- Determine the doubling time for its value. *16 years*
- Determine the annual percent increase in its value. *~5.5%*



14

Match each temperature forecast with its graph.

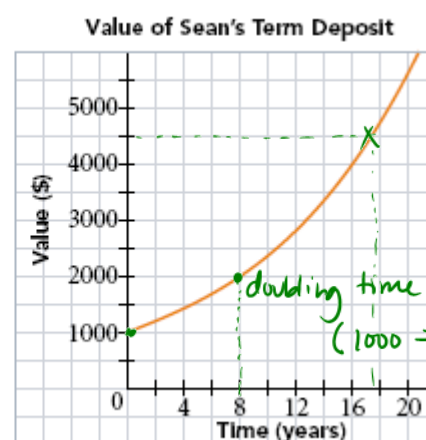
- cool all week
- warm today, but cooling gradually over the next few days
- cold today, but a warm front will arrive tonight



12

Sean has purchased a term deposit that earns compound interest. The value of the term deposit over time is shown in the graph.

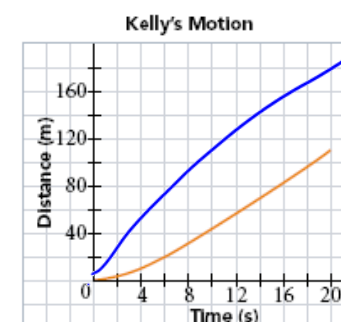
- How much money did Sean deposit? *\$1000*
- When will the term deposit be worth \$4500? *approx. 18 years*
- Determine the doubling time for the deposit. *8 years*
- What is the annual interest rate? *~5.5%*



15

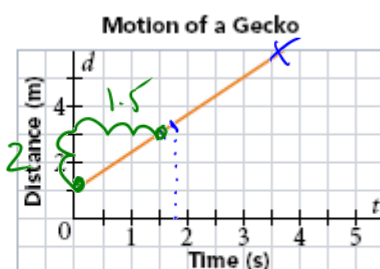
Kelly, a long-distance runner, begins a 1500-m race. Her distance from the starting line is graphed for the first 20 s of the race. Debbie is a sprinter who runs the 200-m race.

- Describe how Debbie's motion would differ from Kelly's in the first 20 s. *Debbie's motion graph would have a steeper slope.*
- Sketch the graph of Kelly's motion on the same axes.



9

A gecko is a small lizard. Students in a grade 9 mathematics class used a CBR to record the motion of a gecko. The gecko slipped out of range of the CBR after 3.5 s. The graph shows the distance of the gecko from the CBR, in metres, over time, in seconds.



- How far was the gecko from the CBR when it slipped out of range? *6 m*

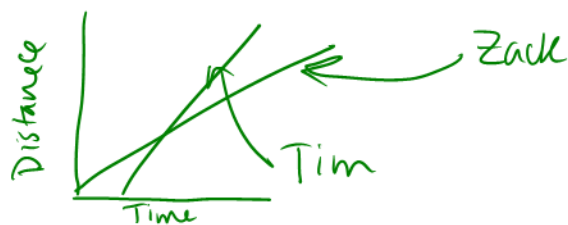
- What was the gecko's speed? *1.33 m/s*

- Write an equation for the gecko's motion. *D = 1.33x + 1*

- When was the gecko 3.2 m away from the CBR? *1.65 s*

10

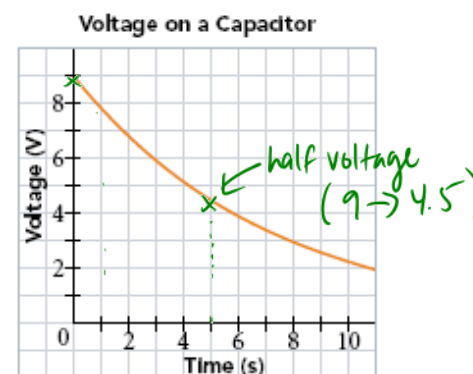
Communication Tim and Zack run a race. Tim always runs faster, so he gives Zack a head-start. Sketch a graph of distance versus time for the boys' race. Explain the important features of your graph.



13

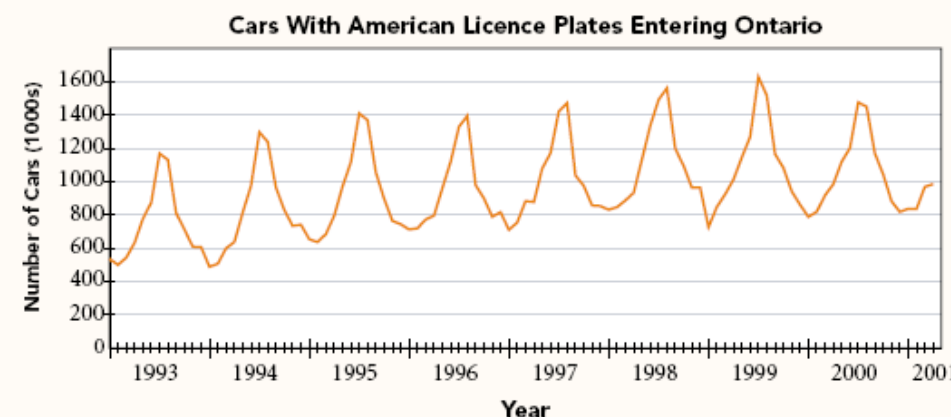
Most VCRs and digital clocks contain a capacitor for power during a brief hydro outage. The graph shows how the voltage on a capacitor decreases with the duration of the hydro outage.

- Estimate the half-voltage; that is, the time needed for the voltage to fall by a factor of 2. *5 seconds*
- What voltage remains after each time? *i) 1 s 8V, ii) 5 s 4.5V, iii) 10 s 2V*
- Estimate the voltage remaining after 0.1 s. *8.9V*



16

Tourists, shoppers, and business people cross the border from the United States into Ontario. The graph shows the number of cars with American licence plates crossing into Ontario from January 1993 to April 2001.



- Describe and explain the short- and long-term trends in the graph.
- Describe the effect of these conditions on the number of American cars entering Ontario:
 - There are no major changes in the economy.
 - The value of the Canadian dollar falls substantially relative to the American dollar.
 - There is a major recession in both countries.
- For each situation in part b, sketch a graph of the number of American cars entering Ontario for 2002–2005.